

Problem 1: Robot Walk Solution

Note that first you will step to either the left or the right, and then up or down. These will keep alternating in the path. Also note that to get back to the starting point, you have to have the same amount of steps to the left as to the right, and the same amount of steps up as steps down.

This means that if we have a path of n steps, we take $n/2$ steps either to the right or left and $n/2$ steps up or down. To return to the start, the steps to right have to be the same amount as steps to the left. So we take $n/4$ steps to the left and $n/4$ steps to the right. We also take $n/4$ steps up and $n/4$ steps down. Note as these have to be the same, n has to be divisible by 4 to have a path that returns to the starting point. So if $n/4 \notin \mathbb{N}$, we get 0 paths.

Now of the $n/2$ steps we take horizontal, $n/4$ have to be to the left. There are $\binom{n/2}{n/4}$ possible ways to take these horizontal steps. The same way, we can argue there are $\binom{n/2}{n/4}$ ways to take the vertical steps. As all ways can be combined, the total amount of paths is

$$\binom{n/2}{n/4}^2.$$